Lotka-Voltera For Developing Countries

"Mathematically, *aid is an initial condition*, while *growth is a behavior of a system* which in most cases has nothing to do with initial conditions"

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Abstract

We study the absolute necessity of "*growth within*" for two economies of two different groups with different growth indices. Growth is simply an attribution of the behavior of a system not what initially the system has or will have, which in this case is an initial condition. It is not the amount of aid given to a particular country that will put the country out of poverty, but the amount of effort they put on **absolute good governance** that will radically change the whole dynamics.

Key words: Growth, competition, cooperation, positive growth

Introduction

I will begin my point with a simple logistic IVP:

$$\begin{cases} \frac{d\psi}{dt} = \beta\psi(t)(K - \psi(t)) \\ \psi(t_0) = \psi_0 \end{cases}$$

Here β is a growth constant for $\psi(t)$ and K is the maximum capacity that $\psi(t)$ can achieve, i.e.

$$K := \psi(\infty) := \lim_{t\to\infty} \psi(t)$$

provided $\beta > 0$ and we clearly see the irrelevancy of the position of ψ_0 so long as it stays being : $0 < \psi_0 < K$.

But if $\beta < 0$, the position of ψ_0 is ephemeral for the behavior of the system.

If $\psi(t_{n_0})$ is so small that the system experiences a catastrophe and suppose an aid is given to lift the state of ψ from $\psi(t_{n_0})$ to a higher state $\psi(t_{n_0}) + \hbar$ which is a new initial state which again is an initial condition and denote by ψ_T the new lifted ψ. Then since $\beta < 0$, $\exists$ a time $h > 0$ such that

$$\psi_T(t_{n_0} + h) = \psi(t_{n_0})$$

i.e., the bad state is coming again and this continues for all time as long as $\psi(t) \searrow$.

Therefore the magnitude of $\hbar$ will not change the behavior of the system which is that of $\psi(t)$.

But for $\beta > 0$, as long as $\psi(t_{n_0}) > 0$, for $t_1 > t_2$ we have $\psi(t_1) \le \psi(t_2)$ and of course $\psi(\infty) = K$ under normal circumstances.

Here β is the most important parameter in the whole dynamical system for $\psi(t)$. In this paper we will consider a mathematical model which is a system of two non-linear first order differential equations for studying two economies of two representative countries form among groups of countries with positive growth indices and with negative growth indices .

As it is well known, the Lotka-Voltera model of systems of non-linear differential equations is used to study how creatures in a certain ecosystem interact, cooperate or compete for coexistence or the elimination of the other. What people should do to control a predator so that the prey shall not disappear. How diseases are transmitted among people who live in one community, how rumors or new ideas spread in among a group of people, etc.

Our main objective here is to see how these models are good for countries with negative indices and countries with positive indices on what should or should not do in order both of them to benefit from their economic relationships, in particular countries with negative growth indices.

The main reason of this work is a plain observation on the failure of donor countries or organizations not to see the mathematical reason behind the complete inability or failure of those countries with negative growth indices not to be able to come out of their constant economic stagnation or simply economic retardation. It has been several decades since the world bank, IMF, wealthy countries donated almost an ever seen amount of money for the so called poor nations, but not a single country came out of the cycle of poverty. They remain constant seekers of help.

What the donors did not see is that the problem is not what they think it is and do not even try to address what the problem is. If by chance you happen to meet somebody who does not know how to catch fish but lives near a river or lake where there are plenty of them. When you go there the person eats well because you catch fish and give him. But when you are not there he will get hungry and needs your or your type so that he gets what he needs.

Here there are two problems which stand out for observation. Your constant presence for his supply and the inability for the person to see what he is capable of. The second problem is the inability for you not to tell him that what you are doing for him is something that the person can do it by himself. There will not be any good example for this than watching people who are living near a big river and lash area waiting for a donation from donor organizations. That is precisely what is happening in most cases between the developing (to be politically correct) and the developed world.

The problem absolutely depends on the type of governance that the developing countries have. Because there is almost an isomorphism between governments of these poor countries, it suffices to take one representative country from these countries and one representative country from the developed ones and study the dynamism that exits between their economic relations and what each of them should do to be better off together.

We denote by $x(t)$ the growth index of the representative country from group I (developed countries) and $y(t)$ be the growth index of the representative country from group II (developing countries). The two variables represent practically the development index (economic, political, social, etc., aspects) of the countries. We assume that these variables change according to the following non-linear system of first order ordinary differential equations.

1 The Lotka-Voltera non-linear Models

As mentioned in the introduction, let $x(t)$ and $y(t)$ denote the growth of two economies with the same potential growth k, but with different growth parameters, whose system of interaction is given by :

$$\begin{cases} \frac{dx}{dt} = \epsilon_1 x (k - x) + \lambda_1 xy \\ \frac{dy}{dt} = \epsilon_2 y (k - y) + \lambda_2 xy \\ x(t_0) = x_0, y(t_0) = y_0 \end{cases}$$

where ϵ_1, ϵ_2 are growth constants with in their own groups and λ_1, λ_2 are growth/competition factors resulted when dealing with the other group. Here we intentionally make the following impositions: $\epsilon_2 < 0$, $\epsilon_1 > 0$.

The explanations for these choices are: Group x is doing well with in itself so that they have a positive growth index within them selves: interpretation: governance here is very good, the system of government is very responsible and accountable. They are globally conscious and the ones at the top echelon of the leadership are determined to lead and take their citizens where the global society is and beyond in a competitive but healthy way.

In the contrary, group y has a negative growth factor ϵ_2: there is absolutely no accountability, no responsibility and lack of sense about the global world where they are part of it. The leadership has no destiny locally and globally other than making a living as ordinary citizens and holding the spot of leadership for no one but themselves. They don't permit competition with in the society and there by from the outside world which in both cases are impediments for growth and development. But the fact of the matter is that they need to have three coordinate systems to look at their developments: the global coordinate system of growth where they stand compared to the outside world, and a local coordinate system to see how locally they do for their development and to use that for their global competition in a particular time, as time is also the third most important factor (or coordinate) they have to take in to account.

The other constants λ_1, λ_2 are just cooperative or competition factors when they are either positive together of negative together. Our objective here is to show that regardless of cooperation or competition that the two groups make, the real and healthy developments are gained on both groups, when both have positive growth index within their types, what they do with in their own group, not only what they gain from the other group.

We will illustrate this phenomenon, by observing what happens to the growth of a group which has a negative growth index while the other has a positive one, where both groups have the same growth potential. We directly go to the analysis of the above system of non-linear first order initial value problems.

The critical points for the system are: $(0,0)\,,(0,k)\,,(k,0)\,,(\widetilde{x},\widetilde{y})$, points where

$$\frac{dx}{dt}=0, \frac{dy}{dt}=0$$

Next we make a phase analysis for these critical points. To do this first lets find the matrix:

$$Df\left(x,y\right)=\begin{bmatrix}\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y}\\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y}\end{bmatrix}_{|(x,y)}$$

$$=\begin{bmatrix}\epsilon_1 k-2\epsilon_1 x+\lambda_1 y & \lambda_1 x\\ \lambda_2 y & \epsilon_2 k-2\epsilon_2 y+\lambda_2 x\end{bmatrix}$$

where,

$$\begin{aligned}f_1\left(x,y\right)&=\epsilon_1 x\left(k-x\right)+\lambda_1 xy\\ f_2\left(x,y\right)&=\epsilon_2 y\left(k-y\right)+\lambda_2 xy.\end{aligned}$$

So,

$$Df\left(0,0\right)=\begin{bmatrix}\epsilon_1 k & 0\\ 0 & \epsilon_2 k\end{bmatrix}$$

As $\epsilon_2<0$, we see that $(0,0)$ is a saddle point with solutions which start out on the y -axis are terminally null and the ones which starts out along the $x-$axis do grow to the ultimate constant k.

Next

$$Df\,(k,0) = \begin{bmatrix} -\epsilon_1 k & \lambda_1 k \\ 0 & \epsilon_2 k + \lambda_2 k \end{bmatrix}$$

Here we choose ϵ_2, λ_2 so that $\epsilon_2 + \lambda_2 > 0$. This will enable us to make this critical point a saddle one.

The next equilibrium point is $(0, k)$ and

$$Df\,(0,k) = \begin{bmatrix} (\epsilon_1 + \lambda_1)\,k & 0 \\ \lambda_2 k & -\epsilon_2 k \end{bmatrix}$$

here all the eigen values are positive and hence the point is unstable (or repeller).

Finally the equilibrium point $(\widetilde{x}, \widetilde{y})$.

$$Df\,(\widetilde{x}, \widetilde{y}) = \begin{bmatrix} \epsilon_1 k - 2\epsilon_1 \widetilde{x} + \lambda_1 \widetilde{y} & \lambda_1 \widetilde{x} \\ \lambda_2 \widetilde{y} & \epsilon_2 k - 2\epsilon_2 \widetilde{y} + \lambda_2 \widetilde{x} \end{bmatrix}$$

Proposition 1 *For $-\epsilon_2 > \lambda_2$, it is possible that both x and y survive but with a constant threat of disappearance for y.*

Corollary 2 *Under the above conditions, the best possibility for y is when the cooperation/competition factor is minimal , i.e. $\lambda_2 \to 0$.*

PROOF. From the system of equations given , an equilibrium point in the first quadrant exists, when $-\epsilon_2 > \lambda_2$ and with a negative slope for

$$\lambda_2 x - \epsilon_2 y + \epsilon_2 k = 0$$

the best possible such a point that lies also on

$$-\epsilon_1 x + \lambda_1 y + \epsilon_1 k = 0$$

is when the line

$$\lambda_2 x - \epsilon_2 y + \epsilon_2 k = 0$$

is almost horizontal and that is when $\lambda_2 \to 0$, or when $\epsilon_2 \to -\infty$ which is in reality impractical.

Is there any mathematical justification that there are times for countries with bad economic shape only to cooperate with others which are better off, not to compete? In other words, is cooperation an option or that at times it has to be done to survive economically? The answer is seen from the following proposition.

Proposition 3 (*When cooperation (not competition) is the only thing to do*)

Under the given conditions, y can not make any competition at all, but to cooperate with x.

PROOF. In order the two groups to have a point of coexistence under these circumstances, there has to be a critical point for the system in the first quadrant which is the intersection point of the straight lines whose equations are given by :

$$-\epsilon_1 x + \lambda_1 y + \epsilon_1 k = 0$$

and

$$\lambda_2 x - \epsilon_2 y + \epsilon_2 k = 0$$

. This point is in the first quadrant if $\frac{-\epsilon_2}{\lambda_2} > 1$. Since $-\epsilon_2 > 0$ we must have $\lambda_2 > 0$. This implies that y can only make cooperation with x.

The following corollary indicates that countries with bad economic shape can in fact be economically eliminated unless they make a timely and correct structural adjustment for their development programs.

Corollary 4 (*The possibility for y to be eliminated*)

When the magnitudes of the cooperation constant and the growth constant within are the same, then group y eventually will disappear. That is , the choices: $\lambda_2 = -\epsilon_2$, is a recipe for eventual disappearance of y it self.

Remark 5 *In most cases of economic failures, the above results, proposition and its corollary are few indicators to the roots of the problems, which are results of lack of proper reasoning, not knowing mathematical formulations of problems and not using mathematical methods to address the problems and eventually making not only bad but wrong deciscions. For instance if they keep on moving on a path in which the path is a mobius band, then they will reach nowhere but to the place or point where they began and in most cases they repeat this process hoping that things will change by miracles which amounts to say a path in a mobius band may be a path in an open growing helix whcih can not happen.*