# ♠ : Plasticity of a Function

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The main purpose of this note is to introduce a concept called **plasticity** for functions.

Particularly I am interested on functions whose graphs have edges or corner points or cusp points and also points where functions are no more differentiable finitely, or which I call points of **no plasticity**.

Given a function $f$ , a domain $D$ and $a \in D$, and assume that the function has one sided derivatives at $a$ :

$f'(a^{+})$: right hand side derivative of the function at $a$ and $f'(a^{-})$ : left hand side derivative of the function at $a$.

**Definition 1** *We say that graph of the function $f$ has* **plasticity** *at $(a, f(a))$ if*

$$|f'(a^{+}) - f'(a^{-})| = 0$$

*else we call the graph* **non plastic** *at the point.*

*We therefore say that $f$ has* **plasticity** *at $a$ if its graph is of* **plasticity** *at $(a, f(a))$.*

**Example 2** *The function $f(x) = x^3$ has* **plasticity** *at $a = 1$, since*

$$|f'(1^{+}) - f'(1^{-})| = |3 - 3| = 0$$

*In fact the function is of* **plasticity** *at ever point of its domain $\mathbb{R}$.*

***Theorem 3*** (Differentiability $\Longrightarrow$ Plasticity) *A function $f$ which is differentiable at $a$ has* **plasticity** *at $a$.*

***Proof.*** *Let $f$ be a function and $a$ be a number in the domain of the function. Assume that $f$ is differentiable at $a$ , that is $f'(a)$ exists finitely.*

*Then both, the left and right sided derivatives of $f$ at $a$ exist and are equal.*

*That is*

$$\begin{aligned} f'\left(a^{+}\right) &= f'\left(a^{-}\right) = f'\left(a\right) \\ &\Rightarrow |f'(a^{+}) - f'(a^{-})| = 0. \end{aligned}$$

$$\therefore f \quad has \quad plasticity \quad at\ a.$$

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**Example 4** *$f(x) = |x|$ is non plastic at $a = 0$, since*

$$|f'(0^{+}) - f'(0^{-})| = 2 \neq 0$$

**Definition 5** *A function $f$ is said to have $\mathfrak{plasticity}$ of order $k$ at $a \in D$ if*

**i)** *the function is $k-$times differentiable at $a$*

**ii)** $|f^{(j)}\left(a^{+}\right) - f^{(j)}(a^{-})| = 0, \forall j = 0, 1, ..., k$

**iii)** $|f^{(k+1)}(a^{+}) - f^{(k+1)}(a^{-})| \neq 0$ *or* $|f^{(k+1)}\left(a^{\pm}\right)| = \infty$

*where $f^{(j)}(a^{\pm})$ are the one sided $j^{th}$ order derivative of $f$ at $a$.*

**Example 6** *The function given by $f\left(x\right) = x^{\frac{7}{3}}$ has plasticity of order $2$ at $0$ but of order $\infty$ at every non-zero real number $a$.*

**Solution 7** *First, the function is continuous at $a = 0$, and thus,*

$$|f^{(0)}\left(0^{+}\right) - f^{(0)}\left(0^{-}\right)| = |f(0) - f(0)| = 0.$$

*Also*

$$|f'(0^{+}) - f'(0^{-})| = |0 - 0| = 0$$

*again,*

$$|f^{(2)}\left(0^{+}\right) - f^{(2)}\left(0^{-}\right)| = 0, but\ f^{(3)}\left(0\right) = \infty.$$

*Therefore, $f$ has plasticity of order $2$ at zero.*

*But at any non-zero real number $a$, we have : $f \in C^{\infty}\left(a\right)$ and therefore*

$$|f^{(k)}\left(a^{+}\right) - f^{(k)}\left(a^{-}\right)| = 0, \forall k = 0, 1, 2, ....$$

*which implies that $f$ has plasticity of order $\infty$ at $a$.*

✠ : **REMARK**: Further development of this concept with few results will follow shortly.